Національний технічний університет України

«Київський політехнічний інститут імені Ігоря Сікорського»

Факультет інформатики та обчислювальної техніки

Кафедра обчислювальної техніки

Методи оптимізації та планування експерименту

Лабораторна робота №6

“ ПРОВЕДЕННЯ ТРЬОХФАКТОРНОГО ЕКСПЕРИМЕНТУ ПРИ ВИКОРИСТАННІ РІВНЯННЯ РЕГРЕСІЇ З КВАДРАТИЧНИМИ ЧЛЕНАМИ”

Виконав:

студент групи ІВ-83

Щебетін Б. Ю.

Перевірив:

ас. Регіда П.Г.

Київ

2020 р.

**Мета:** Провести трьохфакторний експеримент і отримати адекватну модель – рівняння регресії, використовуючи рототабельний композиційний план.

Номер у списку: 22

Варіант завдання: 322





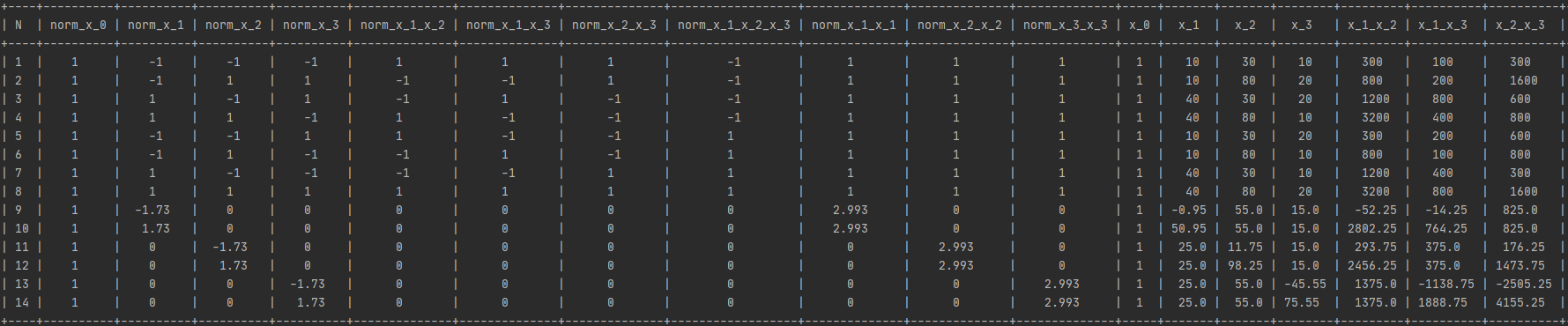
1. Програма

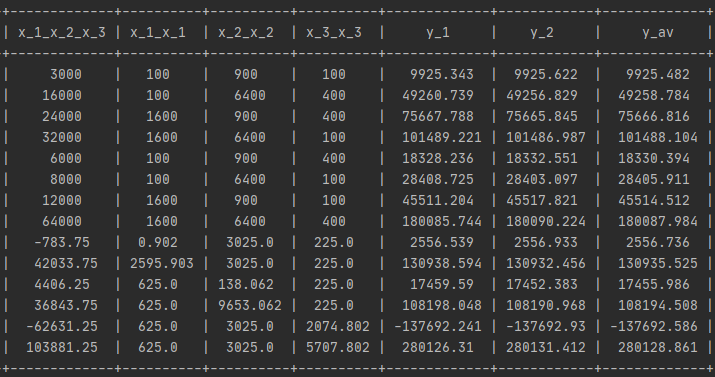
from copy import deepcopy  
from math import sqrt  
from random import random  
  
import numpy as np  
from prettytable import PrettyTable  
  
x1\_min = 10  
x1\_max = 40  
x2\_min = 30  
x2\_max = 80  
x3\_min = 10  
x3\_max = 20  
koefs = [2.1, 1.1, 0.5, 4.2, 1.0, 0.5, 0.9, 2.4, 8.6, 1.0, 2.8] # Koefs for search y  
x\_average\_max = (x1\_max + x2\_max + x3\_max) / 3  
x\_average\_min = (x1\_min + x2\_min + x3\_min) / 3  
y\_max = 200 + x\_average\_max  
y\_min = 200 + x\_average\_min  
  
  
def replace\_column(list\_: list, column, list\_replace):  
 list\_ = deepcopy(list\_)  
 for i in range(len(list\_)):  
 list\_[i][column] = list\_replace[i]  
 return list\_  
  
  
def append\_to\_list\_x(x: list, variant: int):  
 if variant == 1:  
 for i in range(len(x)):  
 x[i].append(x[i][1] \* x[i][2])  
 x[i].append(x[i][1] \* x[i][3])  
 x[i].append(x[i][2] \* x[i][3])  
 x[i].append(x[i][1] \* x[i][2] \* x[i][3])  
 if variant == 2:  
 for i in range(len(x)):  
 x[i].append(x[i][1] \* x[i][2])  
 x[i].append(x[i][1] \* x[i][3])  
 x[i].append(x[i][2] \* x[i][3])  
 x[i].append(x[i][1] \* x[i][2] \* x[i][3])  
 x[i].append(x[i][1] \* x[i][1])  
 x[i].append(x[i][2] \* x[i][2])  
 x[i].append(x[i][3] \* x[i][3])  
 for i in range(len(x)):  
 for j in range(len(x[i])):  
 if round(x[i][j], 3) == 0:  
 x[i][j] = 0  
 x[i][j] = round(x[i][j], 3)  
  
  
def get\_value(table: dict, key: int):  
 value = table.get(key)  
 if value is not None:  
 return value  
 for i in table:  
 if type(i) == range and key in i:  
 return table.get(i)  
  
  
def main(m, n):  
 if n == 14:  
 const\_l = 1.73  
 print(  
 'ŷ = b0 + b1 \* x1 + b2 \* x2 + b3 \* x3 + b12 \* x1 \* x2 + b13 \* x1 \* x3 + b23 \* x2 \* x3 + b123 \* x1 \* x2 \* '  
 'x3 + b11 \* x1 \* x1 + b22 \* x2 \* x2 + b33 \* x3 \* x3')  
 norm\_x = [  
 [+1, -1, -1, -1],  
 [+1, -1, +1, +1],  
 [+1, +1, -1, +1],  
 [+1, +1, +1, -1],  
 [+1, -1, -1, +1],  
 [+1, -1, +1, -1],  
 [+1, +1, -1, -1],  
 [+1, +1, +1, +1],  
 [+1, -const\_l, 0, 0],  
 [+1, const\_l, 0, 0],  
 [+1, 0, -const\_l, 0],  
 [+1, 0, const\_l, 0],  
 [+1, 0, 0, -const\_l],  
 [+1, 0, 0, const\_l],  
 ]  
  
 delta\_x1 = (x1\_max - x1\_min) / 2  
 delta\_x2 = (x2\_max - x2\_min) / 2  
 delta\_x3 = (x2\_max - x3\_min) / 2  
 x01 = (x1\_min + x1\_max) / 2  
 x02 = (x2\_min + x2\_max) / 2  
 x03 = (x3\_min + x3\_max) / 2  
  
 x = [  
 [1, x1\_min, x2\_min, x3\_min],  
 [1, x1\_min, x2\_max, x3\_max],  
 [1, x1\_max, x2\_min, x3\_max],  
 [1, x1\_max, x2\_max, x3\_min],  
 [1, x1\_min, x2\_min, x3\_max],  
 [1, x1\_min, x2\_max, x3\_min],  
 [1, x1\_max, x2\_min, x3\_min],  
 [1, x1\_max, x2\_max, x3\_max],  
 [1, -const\_l \* delta\_x1 + x01, x02, x03],  
 [1, const\_l \* delta\_x1 + x01, x02, x03],  
 [1, x01, -const\_l \* delta\_x2 + x02, x03],  
 [1, x01, const\_l \* delta\_x2 + x02, x03],  
 [1, x01, x02, -const\_l \* delta\_x3 + x03],  
 [1, x01, x02, const\_l \* delta\_x3 + x03],  
 ]  
  
 append\_to\_list\_x(norm\_x, variant=2)  
 append\_to\_list\_x(x, variant=2)  
  
 if n == 8:  
 print(  
 'ŷ = b0 + b1 \* x1 + b2 \* x2 + b3 \* x3 + b12 \* x1 \* x2 + b13 \* x1 \* x3 + b23 \* x2 \* x3 + b123 \* x1 \* x2 \* x3'  
 )  
 norm\_x = [  
 [+1, -1, -1, -1],  
 [+1, -1, +1, +1],  
 [+1, +1, -1, +1],  
 [+1, +1, +1, -1],  
 [+1, -1, -1, +1],  
 [+1, -1, +1, -1],  
 [+1, +1, -1, -1],  
 [+1, +1, +1, +1]  
 ]  
  
 x = [  
 [1, x1\_min, x2\_min, x3\_min],  
 [1, x1\_min, x2\_max, x3\_max],  
 [1, x1\_max, x2\_min, x3\_max],  
 [1, x1\_max, x2\_max, x3\_min],  
 [1, x1\_min, x2\_min, x3\_max],  
 [1, x1\_min, x2\_max, x3\_min],  
 [1, x1\_max, x2\_min, x3\_min],  
 [1, x1\_max, x2\_max, x3\_max]  
 ]  
  
 append\_to\_list\_x(norm\_x, variant=1)  
 append\_to\_list\_x(x, variant=1)  
  
 if n == 4:  
 print('ŷ = b0 + b1 \* x1 + b2 \* x2 + b3 \* x3')  
 norm\_x = [  
 [+1, -1, -1, -1],  
 [+1, -1, +1, +1],  
 [+1, +1, -1, +1],  
 [+1, +1, +1, -1],  
 ]  
 x = [  
 [1, x1\_min, x2\_min, x3\_min],  
 [1, x1\_min, x2\_max, x3\_max],  
 [1, x1\_max, x2\_min, x3\_max],  
 [1, x1\_max, x2\_max, x3\_min],  
 ]  
 if n == 14:  
 y = [[round(sum([koefs[j] \* i[j] for j in range(len(koefs))]) + random() \* 10 - 5, 3) for k in range(m)] for i  
 in x]  
 else:  
 y = np.random.randint(y\_min, y\_max, size=(n, m))  
 # y = np.random.randint(y\_min, y\_max, size=(n, m))  
 y\_av = list(np.average(y, axis=1))  
  
 for i in range(len(y\_av)):  
 y\_av[i] = round(y\_av[i], 3)  
  
 if n == 14:  
 t = PrettyTable(['N', 'norm\_x\_0', 'norm\_x\_1', 'norm\_x\_2', 'norm\_x\_3', 'norm\_x\_1\_x\_2', 'norm\_x\_1\_x\_3',  
 'norm\_x\_2\_x\_3', 'norm\_x\_1\_x\_2\_x\_3', 'norm\_x\_1\_x\_1', 'norm\_x\_2\_x\_2', 'norm\_x\_3\_x\_3', 'x\_0',  
 'x\_1', 'x\_2', 'x\_3', 'x\_1\_x\_2', 'x\_1\_x\_3', 'x\_2\_x\_3', 'x\_1\_x\_2\_x\_3', 'x\_1\_x\_1', 'x\_2\_x\_2',  
 'x\_3\_x\_3'] + [f'y\_{i + 1}' for i in range(m)] + ['y\_av'])  
  
 if n == 8:  
 t = PrettyTable(['N', 'norm\_x\_0', 'norm\_x\_1', 'norm\_x\_2', 'norm\_x\_3', 'norm\_x\_1\_x\_2', 'norm\_x\_1\_x\_3',  
 'norm\_x\_2\_x\_3', 'norm\_x\_1\_x\_2\_x\_3', 'x\_0', 'x\_1', 'x\_2', 'x\_3', 'x\_1\_x\_2', 'x\_1\_x\_3',  
 'x\_2\_x\_3', 'x\_1\_x\_2\_x\_3'] + [f'y\_{i + 1}' for i in range(m)] + ['y\_av'])  
 if n == 4:  
 t = PrettyTable(  
 ['N', 'norm\_x\_0', 'norm\_x\_1', 'norm\_x\_2', 'norm\_x\_3', 'x\_0', 'x\_1', 'x\_2', 'x\_3'] +  
 [f'y\_{i + 1}' for i in range(m)] + ['y\_av'])  
  
 for i in range(n):  
 t.add\_row([i + 1] + list(norm\_x[i]) + list(x[i]) + list(y[i]) + [y\_av[i]])  
 print(t)  
  
 m\_ij = []  
 for i in range(len(x[0])):  
 m\_ij.append([round(sum([x[k][i] \* x[k][j] for k in range(len(x))]) / 14, 3) for j in range(len(x[i]))])  
  
 k\_i = []  
 for i in range(len(x[0])):  
 a = sum(y\_av[j] \* x[j][i] for j in range(len(x))) / 14  
 k\_i.append(a)  
  
 det = np.linalg.det(m\_ij)  
 det\_i = [np.linalg.det(replace\_column(m\_ij, i, k\_i)) for i in range(len(k\_i))]  
  
 b\_i = [round(i / det, 3) for i in det\_i]  
 if n == 14:  
 print(  
 f"\nThe naturalized regression equation: "  
 f"y = {b\_i[0]:.5f} + {b\_i[1]:.5f} \* x1 + {b\_i[2]:.5f} \* x2 + "  
 f"{b\_i[3]:.5f} \* x3 + {b\_i[4]:.5f} \* x1 \* x2 + "  
 f"{b\_i[5]:.5f} \* x1 \* x3 + {b\_i[6]:.5f} \* x2 \* x3 + {b\_i[7]:.5f} \* x1 \* x2 \* x3 + {b\_i[8]:.5f} \* x1 \* x1 + "  
 f"{b\_i[9]:.5f} \* x2 \* x2 + {b\_i[10]:.5f} \* x3 \* x3")  
 if n == 8:  
 print(  
 f"\nThe naturalized regression equation: "  
 f"y = {b\_i[0]:.5f} + {b\_i[1]:.5f} \* x1 + {b\_i[2]:.5f} \* x2 + "  
 f"{b\_i[3]:.5f} \* x3 + {b\_i[4]:.5f} \* x1 \* x2 + "  
 f"{b\_i[5]:.5f} \* x1 \* x3 + {b\_i[6]:.5f} \* x2 \* x3 + {b\_i[7]:.5f} \* x1 \* x2 \* x3")  
 if n == 4:  
 print(  
 f"\nThe naturalized regression equation: "  
 f"y = {b\_i[0]:.5f} + {b\_i[1]:.5f} \* x1 + {b\_i[2]:.5f} \* x2 + {b\_i[3]:.5f} \* x3\n")  
  
 check\_i = [round(sum(b\_i[j] \* i[j] for j in range(len(b\_i))), 3) for i in x]  
 for i in range(len(check\_i)):  
 print(f'ŷ{i + 1} = {check\_i[i]}, y\_av{i + 1} = {y\_av[i]}')  
  
 print("\n[ Kohren's test ]")  
 f\_1 = m - 1  
 f\_2 = n  
 s\_i = [sum([(i - y\_av[j]) \*\* 2 for i in y[j]]) / m for j in range(len(y))]  
 g\_p = max(s\_i) / sum(s\_i)  
  
 table = {2: 0.75, 3: 0.6841, 4: 0.6287, 5: 0.5892, 6: 0.5598, 7: 0.5365, 8: 0.5175, 9: 0.5017, 10: 0.4884,  
 range(11, 17): 0.4366, range(17, 37): 0.3720, range(37, 2 \*\* 100): 0.3093}  
 g\_t = get\_value(table, m)  
  
 if g\_p < g\_t:  
 print(f"The variance is homogeneous: Gp = {g\_p:.5} < Gt = {g\_t}")  
 else:  
 print(f"The variance is not homogeneous Gp = {g\_p:.5} > Gt = {g\_t}\nStart again with m = m + 1 = {m + 1}")  
 return main(m=m + 1, n=n)  
  
 print("\n[ Student's test ]")  
 s2\_b = sum(s\_i) / n  
 s2\_beta\_s = s2\_b / (n \* m)  
 s\_beta\_s = sqrt(s2\_beta\_s)  
 beta\_i = [sum([norm\_x[i][j] \* y\_av[i] for i in range(len(norm\_x))]) / n for j in range(len(norm\_x[0]))]  
 beta\_i = [round(i, 3) for i in beta\_i]  
  
 t = [abs(i) / s\_beta\_s for i in beta\_i]  
 if n == 14:  
 beta\_i = b\_i  
 f\_3 = f\_1 \* f\_2  
 t\_table = {4: 2.776, 5: 2.571, 6: 2.447, 7: 2.365, 8: 2.306, 9: 2.262, 10: 2.228, 11: 2.201, 12: 2.179, 13: 2.160,  
 14: 2.145, 15: 2.131, 16: 2.120, 17: 2.110, 18: 2.101, 19: 2.093, 20: 2.086, 21: 2.08, 22: 2.074,  
 23: 2.069, 24: 2.064, range(25, 30): 2.06, range(30, 40): 2.042, range(40, 60): 2.021, range(60, 100): 2,  
 range(100, 2 \*\* 100): 1.96}  
 d = deepcopy(len(beta\_i))  
 for i in range(len(t)):  
 if get\_value(t\_table, f\_3) > t[i]:  
 beta\_i[i] = 0  
 d -= 1  
 if n == d:  
 n = 8 if n == 4 else 14  
 print(f"n=d\nStart again with n = {n} and m = {m}")  
 return main(m=m, n=n)  
 if n == 14:  
 print(  
 f"\nThe naturalized simplified regression equation: "  
 f"y = {beta\_i[0]:.5f} + {beta\_i[1]:.5f} \* x1 + "  
 f"{beta\_i[2]:.5f} \* x2 + {beta\_i[3]:.5f} \* x3 + {beta\_i[4]:.5f} \* x1 \* x2 + "  
 f"{beta\_i[5]:.5f} \* x1 \* x3 + {beta\_i[6]:.5f} \* x2 \* x3 + {beta\_i[7]:.5f} \* x1 \* x2 \* x3 + "  
 f"{beta\_i[8]:.5f} \* x1 \* x1 + {beta\_i[9]:.5f} \* x2 \* x2 + {beta\_i[10]:.5f} \* x3 \* x3")  
 check\_i = [round(sum(beta\_i[j] \* i[j] for j in range(len(beta\_i))), 3) for i in x]  
  
 if n == 8:  
 print(  
 f"\nThe normalized regression equation: "  
 f"y = {beta\_i[0]:.5f} + {beta\_i[1]:.5f} \* x1 + {beta\_i[2]:.5f} \* x2 + "  
 f"{beta\_i[3]:.5f} \* x3 + {beta\_i[4]:.5f} \* x1 \* x2 + "  
 f"{beta\_i[5]:.5f} \* x1 \* x3 + {beta\_i[6]:.5f} \* x2 \* x3 + {beta\_i[7]:.5f} \* x1 \* x2 \* x3")  
 check\_i = [round(sum(beta\_i[j] \* i[j] for j in range(len(beta\_i))), 3) for i in norm\_x]  
  
 if n == 4:  
 print(  
 f"\nThe normalized regression equation: "  
 f"y = {beta\_i[0]:.5f} + {beta\_i[1]:.5f} \* x1 + {beta\_i[2]:.5f} \* x2 + "  
 f"{beta\_i[3]:.5f} \* x3")  
 check\_i = [round(sum(beta\_i[j] \* i[j] for j in range(len(beta\_i))), 3) for i in norm\_x]  
  
 for i in range(len(check\_i)):  
 print(f'ŷ{i + 1} = {check\_i[i]}, y\_av{i + 1} = {y\_av[i]}')  
  
 print("\n[ Fisher's test ]")  
 f\_4 = n - d  
 s2\_ad = m / f\_4 \* sum([(check\_i[i] - y\_av[i]) \*\* 2 for i in range(len(y\_av))])  
 f\_p = s2\_ad / s2\_b  
 f\_t = {  
 1: [164.4, 199.5, 215.7, 224.6, 230.2, 234, 235.8, 237.6],  
 2: [18.5, 19.2, 19.2, 19.3, 19.3, 19.3, 19.4, 19.4],  
 3: [10.1, 9.6, 9.3, 9.1, 9, 8.9, 8.8, 8.8],  
 4: [7.7, 6.9, 6.6, 6.4, 6.3, 6.2, 6.1, 6.1],  
 5: [6.6, 5.8, 5.4, 5.2, 5.1, 5, 4.9, 4.9],  
 6: [6, 5.1, 4.8, 4.5, 4.4, 4.3, 4.2, 4.2],  
 7: [5.5, 4.7, 4.4, 4.1, 4, 3.9, 3.8, 3.8],  
 8: [5.3, 4.5, 4.1, 3.8, 3.7, 3.6, 3.5, 3.5],  
 9: [5.1, 4.3, 3.9, 3.6, 3.5, 3.4, 3.3, 3.3],  
 10: [5, 4.1, 3.7, 3.5, 3.3, 3.2, 3.1, 3.1],  
 11: [4.8, 4, 3.6, 3.4, 3.2, 3.1, 3, 3],  
 12: [4.8, 3.9, 3.5, 3.3, 3.1, 3, 2.9, 2.9],  
 13: [4.7, 3.8, 3.4, 3.2, 3, 2.9, 2.8, 2.8],  
 14: [4.6, 3.7, 3.3, 3.1, 3, 2.9, 2.8, 2.7],  
 15: [4.5, 3.7, 3.3, 3.1, 2.9, 2.8, 2.7, 2.7, 2.7, 2.7, 2.6, 2.6],  
 16: [4.5, 3.6, 3.2, 3, 2.9, 2.7, 2.6, 2.6],  
 17: [4.5, 3.6, 3.2, 3, 2.8, 2.7, 2.5, 2.3],  
 18: [4.4, 3.6, 3.2, 2.9, 2.8, 2.7, 2.5, 2.3],  
 19: [4.4, 3.5, 3.1, 2.9, 2.7, 2.7, 2.4, 2.3],  
 range(20, 22): [4.4, 3.5, 3.1, 2.8, 2.7, 2.7, 2.4, 2.3],  
 range(22, 24): [4.3, 3.4, 3.1, 2.8, 2.7, 2.6, 2.4, 2.3],  
 range(24, 26): [4.3, 3.4, 3, 2.8, 2.6, 2.5, 2.3, 2.2],  
 range(26, 28): [4.2, 3.4, 3, 2.7, 2.6, 2.5, 2.3, 2.2],  
 range(28, 30): [4.2, 3.3, 3, 2.7, 2.6, 2.4, 2.3, 2.1],  
 range(30, 40): [4.2, 3.3, 3, 2.7, 2.6, 2.4, 2.3, 2.1, 2, 2, 2, 2],  
 range(40, 60): [4.1, 3.2, 2.9, 2.6, 2.5, 2.3, 2.2, 2, 1.9, 1.9, 1.9, 1.9],  
 range(60, 120): [4, 3.2, 2.8, 2.5, 2.4, 2.3, 2.1, 1.9, 1.8, 1.8, 1.8, 1.8, 1.8, 1.8, 1.8, 1.8],  
 range(120, 2 \*\* 100): [3.8, 3, 2.6, 2.4, 2.2, 2.1, 2, 2, 1.9, 1.9, 1.9, 1.8, 1.8]  
 }  
 if f\_p > get\_value(f\_t, f\_3)[f\_4]:  
 n = 8 if n == 4 else 14  
 print(  
 f"fp = {f\_p} > ft = {get\_value(f\_t, f\_3)[f\_4]}.\n"  
 f"The mathematical model is not adequate to the experimental data\n"  
 f"Start again with m = {m} and n = {n}")  
 return main(m=m, n=n)  
 else:  
 print(  
 f"fP = {f\_p} < fT = {get\_value(f\_t, f\_3)[f\_4]}.\n"  
 f"The mathematical model is adequate to the experimental data\n")  
  
  
# n = 14 because if you start with 4 then it will not reach 14  
main(m=2, n=14)

Результати виконання роботи

The mathematical model is not adequate to the experimental data

Start again with m = 2 and n = 14

ŷ = b0 + b1 \* x1 + b2 \* x2 + b3 \* x3 + b12 \* x1 \* x2 + b13 \* x1 \* x3 + b23 \* x2 \* x3 + b123 \* x1 \* x2 \* x3 + b11 \* x1 \* x1 + b22 \* x2 \* x2 + b33 \* x3 \* x3



The naturalized regression equation: y = -13.32900 + 1.50300 \* x1 + 0.70200 \* x2 + 4.70800 \* x3 + 0.99500 \* x1 \* x2 + 0.49100 \* x1 \* x3 + 0.89600 \* x2 \* x3 + 2.40000 \* x1 \* x2 \* x3 + 8.60000 \* x1 \* x1 + 1.00000 \* x2 \* x2 + 2.79900 \* x3 \* x3

ŷ1 = 9926.141, y\_av1 = 9925.482

ŷ2 = 49259.421, y\_av2 = 49258.784

ŷ3 = 75666.011, y\_av3 = 75666.816

ŷ4 = 101487.131, y\_av4 = 101488.104

ŷ5 = 18330.821, y\_av5 = 18330.394

ŷ6 = 28406.741, y\_av6 = 28405.911

ŷ7 = 45514.031, y\_av7 = 45514.512

ŷ8 = 180087.111, y\_av8 = 180087.984

ŷ9 = 2556.22, y\_av9 = 2556.736

ŷ10 = 130935.705, y\_av10 = 130935.525

ŷ11 = 17455.278, y\_av11 = 17455.986

ŷ12 = 108195.248, y\_av12 = 108194.508

ŷ13 = -137694.928, y\_av13 = -137692.586

ŷ14 = 280128.288, y\_av14 = 280128.861

[ Kohren's test ]

The variance is homogeneous: Gp = 0.17047 < Gt = 0.75

[ Student's test ]

The naturalized simplified regression equation: y = -13.32900 + 1.50300 \* x1 + 0.70200 \* x2 + 4.70800 \* x3 + 0.99500 \* x1 \* x2 + 0.49100 \* x1 \* x3 + 0.89600 \* x2 \* x3 + 2.40000 \* x1 \* x2 \* x3 + 8.60000 \* x1 \* x1 + 1.00000 \* x2 \* x2 + 2.79900 \* x3 \* x3

ŷ1 = 9926.141, y\_av1 = 9925.482

ŷ2 = 49259.421, y\_av2 = 49258.784

ŷ3 = 75666.011, y\_av3 = 75666.816

ŷ4 = 101487.131, y\_av4 = 101488.104

ŷ5 = 18330.821, y\_av5 = 18330.394

ŷ6 = 28406.741, y\_av6 = 28405.911

ŷ7 = 45514.031, y\_av7 = 45514.512

ŷ8 = 180087.111, y\_av8 = 180087.984

ŷ9 = 2556.22, y\_av9 = 2556.736

ŷ10 = 130935.705, y\_av10 = 130935.525

ŷ11 = 17455.278, y\_av11 = 17455.986

ŷ12 = 108195.248, y\_av12 = 108194.508

ŷ13 = -137694.928, y\_av13 = -137692.586

ŷ14 = 280128.288, y\_av14 = 280128.861

[ Fisher's test ]

fP = 1.4042533292251047 < fT = 3.1.

The mathematical model is adequate to the experimental data